SOLUTION OF THE KURAMOTO-SIVASHINSKY EQUATION USING SAWI-HOMOTOPY PERTURBATION METHOD

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Abstract

This work presents a hybrid analytical approach for studying the Kuramoto–Sivashinsky (KS) equation by combining the SAWI transform with the Homotopy Perturbation Method (HPM). The KS equation, well known for its nonlinear and dissipative features, poses challenges for obtaining closed-form solutions. In the proposed scheme, the SAWI transform is first used to reformulate the governing equation into a more manageable algebraic expression. The transformed relation is then treated with HPM to generate a rapidly convergent series representation without the need for linearization, mesh generation, or small-parameter assumptions. Symbolic tools in MATLAB were employed to derive the iterative corrections and verify the behavior of the resulting approximation. Comparisons with established exact solutions show that the SAWI–HPM structure yields highly accurate results with very small residual differences. The findings indicate that the proposed hybrid strategy is efficient, easy to implement, and well suited for handling nonlinear evolution equations such as the KS equation.

Keywords: Kuramoto–Sivashinsky equation, SAWI transform, Homotopy Perturbation Method, Partial Differential Equations

Introduction

The Kuramoto–Sivashinsky (KS) equation has attracted considerable attention due to its ability to model a wide range of complex dynamical behaviors, including the movement of flame fronts, instability patterns in thin fluid films, and certain turbulence-driven processes (Zelati et al., 2021). Its formulation encompasses both nonlinear convection and higher-order dissipation, making the equation analytically demanding. These characteristics generally prevent classical solution techniques from yielding exact expressions, which has encouraged the development of various approximate and numerical strategies (Yousif et al., 2014).

Among the tools frequently employed are semi-analytical approaches such as the Variational Iteration Method (VIM) and the Homotopy Perturbation Method (HPM). These methods are known for producing reliable series-type approximations and have often been used for validation when testing newer techniques (Kurulay et al., 2013). In recent years, hybrid procedures that merge integral transforms with perturbation-based iteration have become increasingly popular, as such combinations can improve convergence rates while simplifying the computational workload. One such emerging technique involves the SAWI transform, which has demonstrated remarkable potential in dealing with nonlinear and integro-differential models when used in conjunction with HPM (Saadeh, 2025; Saadeh et al., 2023).

Given that HPM alone has previously produced accurate results for the KS equation (Yousif et al., 2014; Easif, 2013), introducing the SAWI transform into the solution framework offers an opportunity to further enhance accuracy and analytical tractability. The aim of this work is to explore this integrated scheme and assess its effectiveness in capturing the nonlinear and dispersive features that characterize the KS equation. Beyond addressing this particular model, the approach offers a promising pathway for solving broader families of nonlinear differential equations encountered in applied mathematics and physical sciences.

Materials and Methods

The SAWI Transform

The SAWI transform is an integral transform used to simplify differential equations by converting differential operators into algebraic expressions in the transform domain. Let A be the set of functions defined as:

$$A = \left\{ f(t) : \exists\, M, k_1, k_2 > 0 : |f(t)| \le M e^{\frac{|t|}{k_j}} \right\} \quad 1$$

where M is a finite constant, k_1 and k_2 may be finite or infinite. The SAWI transform of a function $f(t)$ denoted by the operator $S[f(t)] = R(\mathrm{v})$, is defined by:

$$S[f(t)] = R(\mathrm{v}) = \frac{1}{v^2}\int_0^\infty f(t) e^{-\frac{t}{v}} dt, \quad t \ge 0,\ k_1 \le v \le k_2 \quad 2$$

Inverse SAWI Transform

The Inverse SAWI transform is given by:

$$\mathrm{S}^{-1}[R(\upsilon)] = \mathrm{f(t)} = \frac{1}{2\pi \mathrm{i}} \int_{u-\iota\infty}^{u+\iota\infty} \upsilon e^{\upsilon t} R(\upsilon) d\upsilon \quad 3$$

Sawi Transform of the Derivatives of the Function F(t)

Let $R[v]$ SAWI transform of $[S[f(t)] = R[v]$ then:

$$S[f'(t)] = \frac{R(v)}{v} - \frac{f(0)}{v^2} \quad 4$$

$$S[f''(t)] = \frac{R(v)}{v^2} - \frac{f'(0)}{v^2} - \frac{f(0)}{v^3} \quad 5$$

$S[f^n(t)] = \frac{R(v)}{v^n} - \sum_{k=0}^{n-1} \frac{f^k(0)}{v^{n-k+1}}$ 6

Table 1: SAWI transform of Basic Functions

$f(t)$	$S[f(t)] = k(v)$
k	$\frac{k}{v}$
t	1
t^n	v^{n-1}
e^{at}	$\frac{1}{v(1-av)}$
$sinkt$	$\frac{k}{(1+k^2v^2)}$
$coskt$	$\frac{1}{v(v^2+k^2)}$
$sinhkt$	$\frac{k}{(1-k^2v^2)}$
$cosh\,kt$	$\frac{k}{v(1-k^2v^2)}$

Homotopy Perturbation Method (HPM)

Consider a general nonlinear equation:
$L(u) + \mathrm{N}(\mathrm{u}) = 0,$ 7
where L is a linear operator and N a nonlinear operator. A convex homotopy is constructed as:

$H(u,p) = (1-p)[L(u) - L(u_0)] + p[L(u) + N(u)] = 0$ 8

where $p \in [0,1]$ is the embedding parameter, and u_0 is an initial approximation that satisfies the initial or boundary conditions.

When $p = 0$:

$H(u,0) = L(u) - L(u_0) = 0$ 9

and when $p = 1$:

$H(u,1) = L(u) + N(u) = 0$ 10

which recovers the original equation (7)

HPM assumes the solution as a power series in :

$u = u_0 + pu_1 + p^2u_2 + p^3u_3 + \cdots$ 11
and at $p \to 1$, the approximate solution becomes

$u = u_0 + u_1 + u_2 + \cdots$ 12

He's polynomials

For nonlinearities of the form $N(u)$, He's polynomials are introduced to systematically expand the nonlinear term:

$N(u) = \sum_{n=0}^{\infty} p^n H_n,$

where

$H_n(u_0, \dots u_n) = \frac{1}{n!}\frac{\partial^n}{\partial p^n}\left(N(\sum_{n=0}^{\infty} p^n H_n)\right)_{p=0}$ 13

APPLICATION OF SAWI-HOMOTOPY PERTURBATION METHOD TO KURAMOTO-SIVASHINSKY EQUATION

Consider the Kuramoto Sivashinsky equation:

$u_t + uu_x + \alpha u_{xx} + \gamma u_{xxx} + \beta u_{xxxx} = 0$ 14
with the initial condition:

$u(x,0) = f(x), \quad a \le x \le b$ 15
Define the SAWI transform of the solution as:
$R(x,v) = \mathcal{S}[u(\mathrm{x},\mathrm{t})]$
Applying the SAWI transform to Eqn (14)

$\mathcal{S}[u_t] + \mathcal{S}[uu_x] + \mathcal{S}[\alpha u_{xx}] + \mathcal{S}[\gamma u_{xxx}] + \mathcal{S}[\beta u_{xxxx}] = 0$

$\frac{R(x,v)}{v} - \frac{f(x)}{v^2} + \mathcal{S}[uu_x] + \mathcal{S}[\alpha u_{xx}] + \mathcal{S}[\gamma u_{xxx}] + \mathcal{S}[\beta u_{xxxx}] = 0$ 16
Solving for $R(x,v)$

$R(x,v) = \frac{f(x)}{\mathrm{v}} - v\mathcal{S}[uu_x] - v(\alpha u_{xx} + \gamma u_{xxx} + \beta u_{xxxx})$ 17

The solution in the physical domain is recovered through the inverse SAWI transform:

$u(x,t) = \mathcal{S}^{-1}[R(x,v)]$ 18
To apply HPM, we define

$L(u) = u_{\mathrm{t}},\ \ \mathrm{N}(\mathrm{u}) = \mathrm{u}u_x - \alpha u_{xx} + \gamma u_{xxx} + \beta u_{xxxx}$

The KS equation becomes:

$L(u) + \mathrm{N}(\mathrm{u}) = 0$

We construct the homotopy:

$H(u,p) = (1-p)[u_{\mathrm{t}} - u_{0,\mathrm{t}}] + p[u_{\mathrm{t}} + N(u)] = 0$ 19

with $u_0(x,t) = f(x)$.

The HPM series solution is written as:

$u = u_0 + \mathrm{p}u_1 + \mathrm{p}^2u_2 + \mathrm{p}^3\mathrm{u}_3 + \cdots$

Substituting into (19) and equating coefficients of like powers of p yields:
$u_0(x,t) = f(x)$ 20
$u_1(x,t) = -\mathcal{S}^{-1}[v\mathcal{S}(H_0 + \alpha u_{0,\mathrm{xx}} + \gamma u_{0,\mathrm{xxx}} + \beta u_{0,\mathrm{xxxx}})]$ 21
$u_2(x,t) = -\mathcal{S}^{-1}[v\mathcal{S}(H_1 + \alpha u_{1,\mathrm{xx}} + \gamma u_{1,\mathrm{xxx}} + \beta u_{1,\mathrm{xxxx}})]$ 22
and in general:
$u_m(x,t) = -\mathcal{S}^{-1}[v\mathcal{S}(H_{m-1} + \alpha u_{\mathrm{m-1,xx}} + \gamma u_{\mathrm{m-1,xxx}} + \beta u_{\mathrm{m-1,xxxx}})]$ 23
The approximate solution obtained is then written as

$u(x,t) = u_0 + u_1 + u_2 + \cdots$ 24

Results and Discussion

Example 1: Consider the nonlinear differential problem (Mehdi & Ghanbari, 2010):

$$u_t + uu_x + u_{xx} + u_{xxxx} = 0 \quad (25)$$

with initial condition:

$$u(x,0) = c + \frac{5}{19}\sqrt{\frac{11}{19}}\left[11\tanh^3\big(k(x-x_0)\big) - 9\tanh\big(k(x-x_0)\big)\right] \quad (26)$$

The exact solution for the problem is:

$$u(x,t) = c + \frac{5}{19}\sqrt{\frac{11}{19}}\left[11\tanh^3\big(k(x-ct-x_0)\big) - 9\tanh\big(k(x-ct-x_0)\big)\right] \quad (27)$$

Table 2: Numerical Comparison at $t = 1$

x	SHPM Solution	Exact Solution	Absolute Error
0.000000	0.499756	0.499727	0.000029
0.500000	0.499835	0.499855	0.000020
1.000000	0.499987	0.499961	0.000026
1.500000	0.499897	0.500049	0.000151
2.000000	0.500061	0.500121	0.000060
2.500000	0.500232	0.500181	0.000051
3.000000	0.500272	0.500230	0.000042
3.500000	0.500238	0.500271	0.000033
4.000000	0.500366	0.500305	0.000061
0.000000	0.499756	0.499727	0.000029
4.500000	0.500305	0.500333	0.000028
5.000000	0.500372	0.500356	0.000016

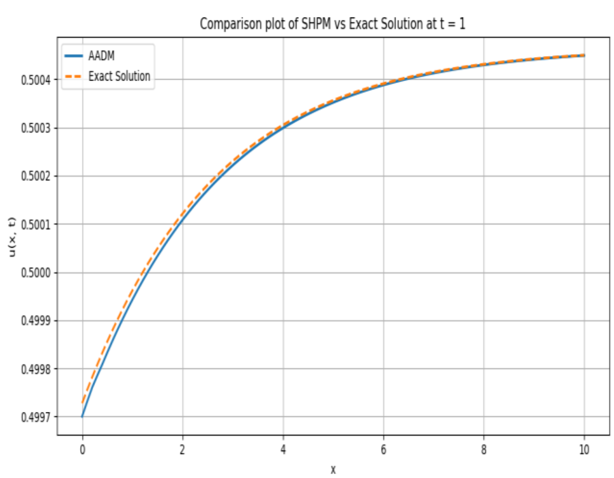

Figure 1: Numerical Comparison at $t = 1$

Discussion

The numerical results show that the SAWI–HPM solution is in excellent agreement with the exact solution over the entire spatial domain. The absolute errors are very small, generally of order of 10^{-5}, and no spurious oscillations or numerical artifacts are observed. Figure 1, which plots both the SAWI–HPM and exact profiles, reveals that the two curves almost completely overlap. This indicates that the hybrid method can accurately capture the waveform structure associated with the KS dynamics, including both dispersive and dissipative effects.

Example 2: Consider the nonlinear differential problem (Mehdi & Ghanbari, 2010)

$$u_t + uu_x - u_{xx} + u_{xxxx} = 0 \quad (28)$$

with initial condition:

$$u(x,0) = c + \frac{5}{19\sqrt{19}}\left[11\tanh^3\big(k(x-x_0)\big) - 3\tanh\big(k(x-x_0)\big)\right] \quad (29)$$

The exact solution is:

$$u(x,t) = c + \frac{5}{19\sqrt{19}}\left[11\tanh^3\big(k(x-ct-x_0)\big) - 3\tanh\big(k(x-ct-x_0)\big)\right] \quad (30)$$

Table 3: Numerical Comparison at $t = 1$

x	SHPM Solution	Exact Solution	Absolute Error
0.000000	0.582667	0.582702	0.000035
0.500000	0.582705	0.582750	0.000046
1.000000	0.582795	0.582790	0.000005
1.500000	0.582756	0.582823	0.000067
2.000000	0.582807	0.582850	0.000043
2.500000	0.582882	0.582873	0.000009
3.000000	0.582891	0.582892	0.000001
3.500000	0.582891	0.582907	0.000016
4.000000	0.582926	0.582920	0.000007
4.500000	0.582917	0.582930	0.000013
5.000000	0.582939	0.582939	0.000000

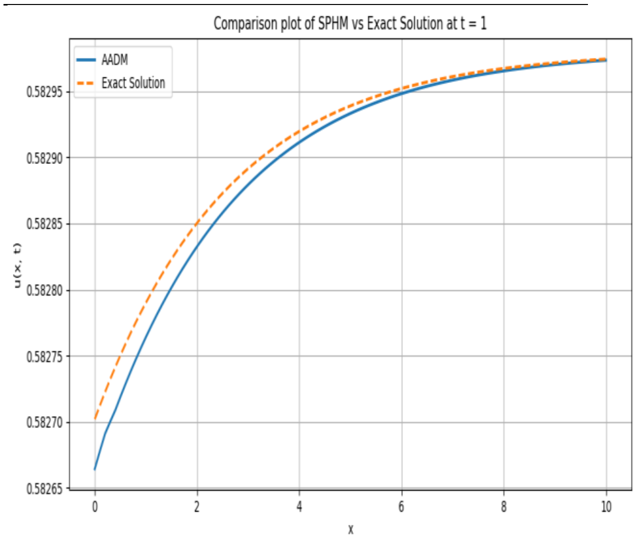

Figure 2: Numerical Comparison $t = 1$

Discussion

The absolute errors are extremely small, frequently of order of 10^{-5}or smaller and vanishing at some grid points. The curves in Figure 2 show nearly perfect overlap, confirming the high accuracy and robustness of the SAWI–HPM method for this second variant of the KS equation. Taken together, the two examples demonstrate that the proposed approach remains stable and efficient across different forms of KS-type models.

Conclusion

This work introduced a combined analytical approach based on the SAWI transform and the Homotopy Perturbation Method for studying the Kuramoto–Sivashinsky equation. The method offers a straightforward pathway for handling nonlinear evolution equations because it avoids common complications such as discretization, linearization, or the introduction of perturbation parameters. By transforming the governing equation into a more tractable form and then applying a homotopy-based series expansion, the approach delivers rapidly converging approximations. The test problems examined in this study demonstrate that the SAWI–HPM framework is capable of reproducing known exact solutions with a high degree of accuracy. The computed errors were consistently small, and the approximate solutions closely matched the analytical profiles across the entire solution domain. These observations confirm that the method is both stable and reliable for equations characterized by strong nonlinearity and higher-order derivatives. Overall, the findings suggest that combining the SAWI transform with HPM provides a powerful and flexible tool for addressing nonlinear partial differential equations. With its good convergence behavior and ease of implementation, the technique has the potential to be extended to a wide range of models encountered in fluid dynamics, applied mathematics, and related scientific fields.

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